On the possibility of determining the focal parameters of an earthquake using a local network equipped with three-component seismometers

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Summary

By exploiting the possibility of determining the angle of emergence and the azimuth of a seismic ray at an observation point equipped with one vertical seismograph and two horizontal ones and using recent digital data acquisition techniques, an effective method for locating the focus of a seismic event is proposed. By means of this method it is possible to obtain, in addition to the point customarily considered as the "barycentre" of the focal volume, also a quantity which represents the "convergence" of various seismic rays on the source zone.

This method makes it possible actually to "determine" the focal parameters and does away with the need for approximation methods of the least squares type.

In the present investigation, tests have been carried out to determine the accuracy of the angle of emergence and the azimuth for a number of earthquakes, together with other theoretical tests for the relocation of the focal parameters using a three-dimensional crust model.

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RIASSUNTO

Sfruttando la possibilità di determinare l'angolo di emergenza e l'azi-
mut di un raggio sismico in un punto di osservazione provvisto di un
sismografo verticale e dei due orizzontali (terna), avvalendosi di recenti
tecniche di acquisizione dati in digitale, è possibile proporre un metodo
efficace per la locazione del fuoco di un evento sismico. Il metodo con-
sente di ottenere, oltre al consueto punto inteso come « baricentro » del
volume focale, una grandezza che rappresenta la « convergenza » dei vari
raggi sismici sulla sorgente.

Questo metodo permette una vera e propria « determinazione » dei
parametri focali fornendo la possibilità di svincolarci da tecniche di stima
per approssimazioni successive del tipo minimi quadrati.

Nel presente lavoro sono state eseguite delle prove per verificare la
precisione dell'angolo di emergenza e dell'azimut su un certo numero di
terremoti nonché alcuni test teorici per la rilocazione dei parametri focali
utilizzando un modello di crosta tridimensionale.

INTRODUCTION

The problem of the space-time location of an earthquake's
focus generally takes the form of a search for a point having
coordinates x, y, z, t to which a physical significance may be
assigned. This approach becomes a serious obstacle when it comes
to attempting to re-establish the connection between the geolo-
gical aspects and the physico-mathematical ones required for a
description of the source of a seismic event.

It has been found in practice that the space coordinates
of the focus alone may be determined from the data obtained
from a single observation site provided that the value of the
ground displacement of the longitudinal wave impinging on the
earth's surface is known accurately enough. In other words, the
records of the three components of a seismic station must show
the beginning of the longitudinal and transverse wave sharply.
If the ground displacement of the first impulse on the East-
West and North-South components are denoted as $\bar{X}_E$ and X_N,

respectively, the apparent azimuth A, is determined by the following expression

$$A = \text{arctang } \frac{X_E}{X_N} \qquad [1]$$

Then, if sensitive accurate instruments are available also the value of the focus z may be determined. Denoting the ground displacement on the vertical component as X_z, the apparent angle of emergence is obtained from the equation:

$$\bar{e} = \text{arctang } \frac{X_z}{\sqrt{X^2_N + X^2_E}} \qquad [2]$$

The reliability obviously depends on the purity of the P wave considered.

Lastly, the theory of elasticity gives a relationship between the apparent angle of emergence and the true angle e:

$$\sin \bar{e} = 1 - 2 \left(\frac{V_{s_i}}{V_{p_i}}\right)^2 \cos^2 e \qquad [3]$$

This approach, which has in the past been successfully used by all seismic observatories to locate an earthquake's focus, i.e. the starting point of the propagation of a perturbation caused by the sudden breakdown in the equilibrium under the earth's crust, now allows us to introduce a method for defining the source also in cases that cannot be trated as a mere geometric point.

The french seismologist Montessus de Ballore in 1896 wrote:

"La dètermination du foyer ou hypocentre d'un tremblement de terre et de son épicentre est un des problèms capitaux de la seismologie, car de sa solution dépendront les considérations géologiques à faire intervenir relativement à la genése du phénoméne.

Malheureusement, il s'en faut de beaucoup que l'on soit arrivé à une méthode sûre; mais peut-être que cette difficulté est insurmontable en ce sens que l'on chercha, par la géometrie on par le calcul, un point d'où l'on suppose émané le mouvement séismique, alors qu'en réalité il s'agit d'une ligne, d'une surface, voire même d'un volume de dimension non négligeables".

This problem was in fact solved in the search for a point that, at a pre-established time, generates an initially spherical, isotropic, progressive condensation wave; the propagation of this disturbance takes place in an elastic, homogeneous, isotropic and suitably stratified medium.

Let us suppose that the wave front incident on the surface is recorded by a network of seismological stations equipped with the three ground displacement components at each recording site. The value of the two angles, together with the arrival times of the first impulse at each station in the network, make it possible to describe the previous behaviour of the longitudinal wave to compute, in new way, the hypocentre of an earthquake.

This kind of approach could prove to be a useful tool for studying the seismic activity of geological structures.

EXPERIMENTAL DETERMINATION OF THE ANGLES

During the seismic period affecting the Friuli in 1976, the mobile station of the Istituto Nazionale di Geofisica, equipped with a three-component seismometer and set up in the epicentral zone, recorded hundreds of aftershocks on analogic magnetic tape. Some tens of events were subsequently converted into digital form at a sampling rate of 150 cps on four channels (the three components plus time) and memorized on computer-compatible

tape. Seven of these events displaying an impulse beginning under differing ground noise conditions were processed.

For each component of each earthquake was calculated the ratio between, the sum of the first 100 points of the condensation wave (and any of its reflections) and an equal number of points of the noise contained in the recording was made using the ground displacement module. The value thus obtained gives the ratio between seismic signal and ground noise.

The seismometers used were of the Geotech S-13 type with a natural period of one second under critical damping conditions. In such a case, the dynamic amplification of a seismometer is obtained using the following expression:

$$V = \frac{U}{1 + u^2} \qquad [5]$$

where V is the dynamic amplification, U the static amplification and u the ratio between the period of the seismic wave being examined and the natural period of the seismometer.

For the compression waves of the events examined $u \to 0$ so that the dynamic amplification tends toward the static amplification value, which is the same for all three components of the mobile station.

Using equations [1], [2] and [3], the mean values of the angle of emergence and the azimuth were determined for the first 10 points successive to the arrival time of the first impulse.

The standard deviations for the two angles are shown in Fig. 1 as a function of the signal/noise ratio for each seismic event.

It can be seen how the errors affecting the angles rapidly tend towards zero for signal/noise ratio values exceeding 4.
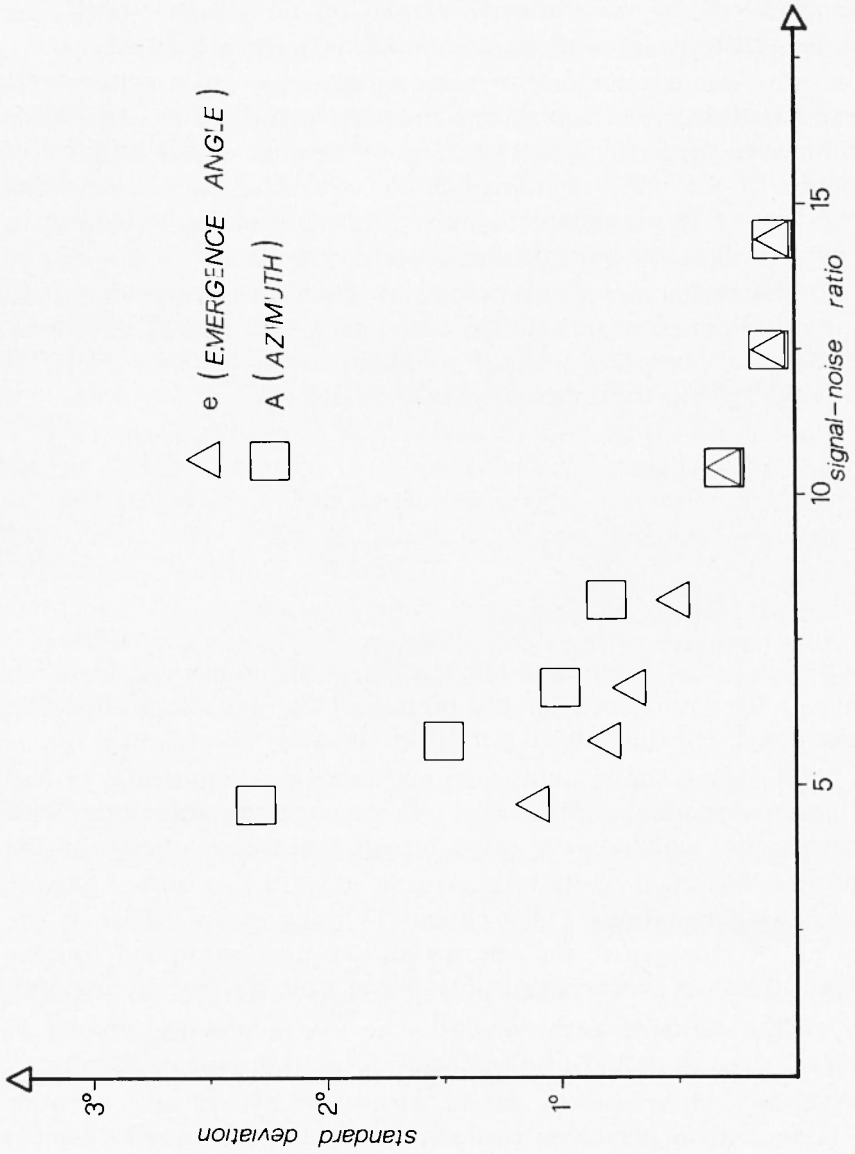
Fig. 1 - Standard deviations of angles of emergence and azimuth expressed in degrees, as a function of signal/noise ratio.

DEFINITION OF THE FOCAL PARAMETERS

The earthquake focus is normally determined from the values of the arrival times of direct and refracted longitudinal waves. The only necessary condition for a solution to be found is that at least four first arrival observations be known in four observation points. In practice, however, such a condition is mostly found to be insufficient. The times read off the seismograms are affected by errors and the travel time diagrams used may not reflect the true velocities of the longitudinal waves in the case of the profiles under examination.

If the number of first arrival readings is greater than four, computing programs based on least squares type approximation techniques are generally used.

In this case it is possible to assign to the solution a statistical error to allow its reliability to be evaluated with regard to experimental data scatter.

The solution is obtained by assigning space-time coordinate values to a starting point using specified criteria. Then, by means of successive corrections of these values, the deviation between theoretical and experimental arrival times at the various seismic stations is reduced to a minimum.

Unfortunately, there are serious drawbacks to this method. Generally speaking, starting from different points leads to different results being obtained. The latter are, in any case, heavily dependent on the type of crust model used.

As we have seen, an observation point equipped with a three-component seismometer can be used to obtain, in addition to the arrival time of the first impulse, also the values of two angles from which the ground displacement of the condensation wave can be determined. Of course, there is an increase in the overall amount of information. This allows us to make substantial modifications to the method used to locate the focus of a seismic event. It is, of course, possible to make an estimate, i.e. to start from a trial point and gradually correct to reduce the residues, which in this case consist of the differences between the theoretical and experimental values of the times and angles. In this

paper, a different method has been proposed, by means of which it is possible to define not only a point but also a quantity linked to the convergence of the various seismic ray on the source zone.

The method consists of tracing back the path followed by each ray using a three-dimensional crust model (taking into consideration also the lateral heterogeneities). The model is in fact made up of various sized suitably stratified "blocks" of various sizes, each having a different velocity. In passing through different "blocks", the seismic rays are subject to the laws of optical geometry. In order to take into account the error in each angle, four rays and not just one, are generally used for each seismological station. They form the vertices of a pyramid, the directions of which are defined by $A+dA$, $e+de$; $A+dA$; $e-de$; $A-dA$, $e+de$; $A-dA$, $e-de$, where $2dA$ and $2de$ are the standard deviations of the azimuth and the angle of emergence, respectively. As we have seen, they are dependent on the signal/noise ratio of each seismic record.

The first step was to follow the various seismic ray of each observation site until the time in which the station nearest the source recorded the first shock. From this time on, time intervals, dt of 0.05 seconds were used (a value suggested by practical considerations). For each dt there is generally a different spatial distribution of the points representing the wave front at different points in time.

The "volume" obtained from each distribution will tend to decrease on approaching the source, and in general to increase and diverge as it moves away from it. Steps were then taken to determine the configuration corresponding to the "minimum volume" at a certain point in time representing the origin time of the seismic event and its "barycentre".

In addition to the value of this "volume", which obviously depends on the errors associated with the times and angles, as well as on the model used, it is also possible to find a quantity which represents the "convergence" of the various rays on the source zone.

It is obtained by calculating the sum the minimum distances between the "barycentre" of the focal zone and each seismic ray.

RESULTS OBTAINED USING A SIMULATION

In order to test the proposed method, a series of simulations were carried out to obtain values for the focal parameters previously introduced for a certain number of seismic events.

TABLE 1

1º layer						3º layer				
3.86	4.28	4.57	4.74	4.78		4.45	4.35	4.28	4.21	4.21
3.86	4.25	4.35	4.64	4.74		4.57	4.42	4.35	4.28	4.21
3.86	4.35	4.35	4.50	4.57		4.71	4.50	4.39	4.35	4.28
3.64	3.86	4.07	4.28	4.35		4.78	4.64	4.50	4.39	4.35
3.50	3.64	3.86	4.07	4.07		4.78	4.71	4.64	4.57	4.39
		Z = .4 Km						Z = 1.6 Km		
2º layer						4º layer				
4.78	4.78	4.78	4.82	4.82		4.57	4.74	4.85	4.92	4.92
4.74	4.78	4.78	4.78	4.82		4.50	4.57	4.74	4.85	4.85
4.71	4.74	4.78	4.78	4.78		4.35	4.42	4.57	4.74	4.74
4.67	4.71	4.74	4.78	4.78		4.28	4.28	4.24	4.57	4.64
4.67	4.67	4.71	4.74	4.74		4.28	4.28	4.35	4.50	4.57
		Z = 1.0 Km						Z = 2.0 Km		

Velocities of initial model: values arranged in order of successive layers and the velocities of each layer are order in plan view; each rectangle measures 2x3 km.
1st layer 0.4 km thickness
2nd layer 1.0 km   »
3rd layer 1.6 km   »
4th layer 2.0 km   »
5th layers is 5.10 km/sec throughout. thickness is 5.0 km all speeds are in km/sec.

A "block" model (tab. 1 and fig. 2) with different velocities
was defined over an area of 10 km x 15 km. Eight observation

TABLE 2

x	y	z
7.5	17.5	6.
7.5	15.5	2.5
7.5	13.5	1.
7.5	11.5	0.5
7.5	9.5	5.
10.5	17.5	2.5
10.5	15.5	4.
10.5	13.5	3.5
10.5	11.5	1.
10.5	9.5	2.5
13.5	17.5	4.
13.5	15.5	5.5
13.5	13.5	3.
13.5	11.5	2.
13.5	9.5	3.5
16.5	17.5	3.
16.5	15.5	1.5
16.5	13.5	4.
16.5	11.5	5.
16.5	9.5	0.5
19.5	17.5	2.
19.5	15.5	2.5
19.5	13.5	0.5
19.5	11.5	2.
19.5	9.5	4.5

Coordinates of initial hypocentres.
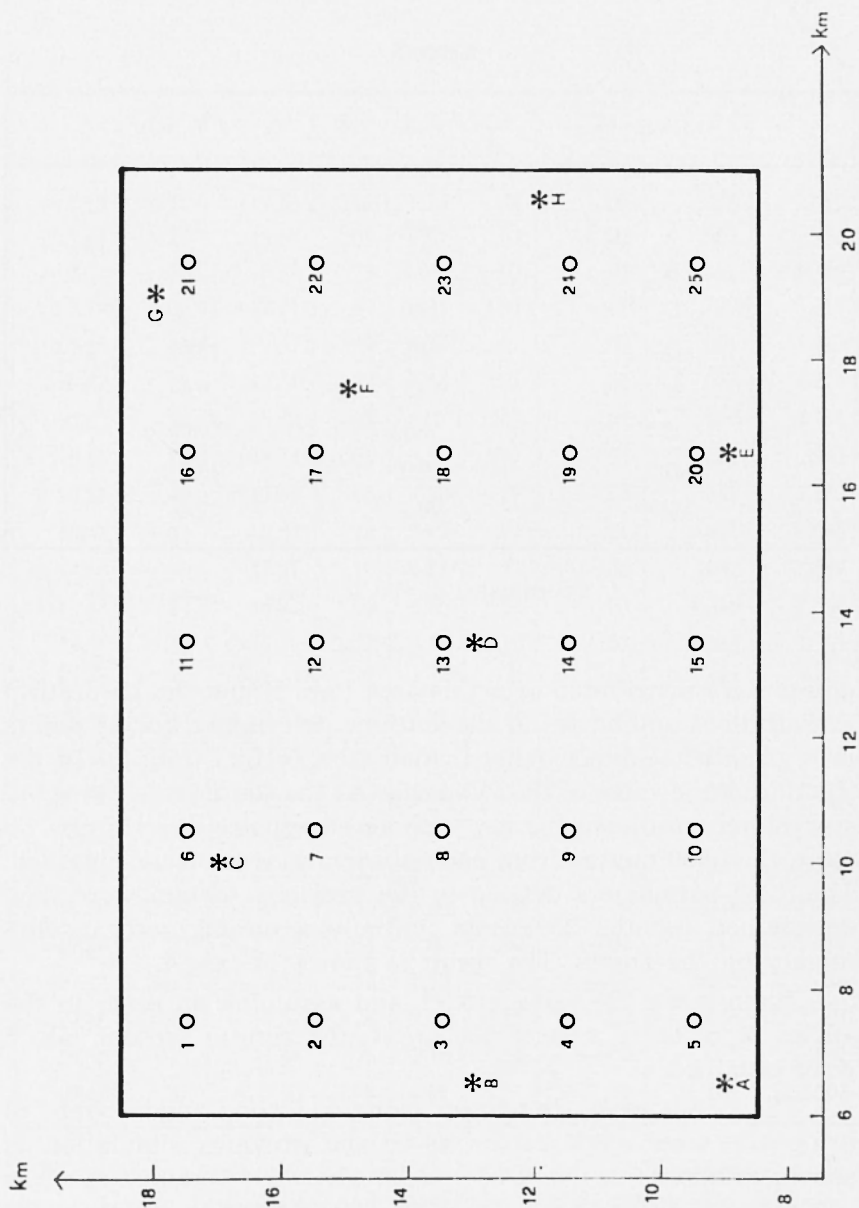
Fig. 2 - Map of simulation zone; circles indicate the epi-
centres, asterisks the stations.

TABLE 3

station	X	Y
A	6.5	9.0
B	6.5	13.0
C	10.0	17.0
D	13.5	13.0
E	16.5	9.0
F	17.5	15.0
G	19.0	18.0
H	20.5	12.0

Coordinates of the stations.

points were distributed over this area (tab. 3) and the theoretical arrival times and angles at the stations determined for 25 points in a given three-dimensional distribution of foci (tab. 2). In the simulation the value of the two angles at the source was corrected successively until seismic ray with an emergence differing by no more than 500 metres from each observation site were obtained. The focal parameters defined in the previous section were then determined for the 25 events, initially assuming zero indeterminacy on the angles. The result is shown in tab. 4.

Again using the same model, and assuming an error in the angles of 5° for e and of 10° for A, the results set out tab. 5 were obtained.

The result of a simulation using a second model (tab. 6) using the same angle errors as in the previous simulation is shown in tab. 7.

For the sake of comparison, the values obtained using only arrival times and based on a least squares technique are shown in tab. 8.

TABLE 4

X	Sx	Y	Sy	Z	Sz	To	V	C
7.18	.51	17.53	.15	6.10	.34	—.05	.213	2.459
7.64	.13	15.59	.09	2.57	.03	—.03	.002	.524
7.57	.30	13.59	.21	.99	.02	.01	.011	1.760
7.78	.06	12.21	.89	2.35	1.31	—.17	.739	5.311
7.55	.13	9.47	.09	4.99	.01	.00	.001	.413
10.57	.29	17.54	.22	2.48	.01	.01	.003	1.224
10.57	.29	15.30	.47	4.07	.09	.01	.096	2.191
10.43	.14	13.53	.03	3.52	.01	—.01	.000	.741
10.57	.40	11.88	.58	1.88	.87	—.03	1.648	7.883
10.46	.22	9.62	.19	2.49	.01	.01	.002	1.048
13.54	.30	17.57	.19	4.21	.13	—.04	.062	1.549
13.68	.34	15.72	.91	5.66	.51	—.03	1.246	5.168
13.58	.32	13.51	.26	3.13	.10	—.02	.068	1.901
13.57	.19	11.63	.48	1.95	.59	.02	.442	2.184
13.44	.13	9.51	.16	3.41	.11	—.00	.019	.781
16.54	.24	17.51	.05	3.05	.01	—.02	.001	.618
16.51	.34	15.36	.15	1.49	.01	.00	.003	1.122
16.52	.31	13.65	.31	3.66	.05	.09	.037	1.516
16.44	.07	11.15	.33	4.95	.19	—.04	.038	1.254
16.56	.24	9.54	.15	.58	.09	.00	.026	1.196
19.56	.23	17.53	.31	2.07	.02	—.02	.010	1.416
19.32	.15	15.53	.12	2.52	.00	—.01	.000	.618
19.48	.22	13.51	.24	.63	.15	.02	.063	1.701
19.58	.11	11.64	.26	2.16	.11	—.02	.025	.841
19.41	.32	9.53	.35	4.31	.21	.01	.184	2.002

Focal parameter values assuming zero indeterminacy
at the two angles and using the initial model. Sx, Sy,
Sz, are the semi-amplitudes of the focal zone, V is the
volume and C is a quantity related to ray convergence
on the focal zone.

TABLE 5

X	Sx	Y	Sy	Z	Sz	To	V	C
7.53	.83	17.51	.97	5.72	.61	.10	3.919	2.583
7.54	.62	15.63	.54	2.38	.86	—.03	2.320	1.017
7.55	.76	13.61	1.25	3.38	1.83	—.019	13.751	11.389
7.99	.71	12.20	1.74	2.68	1.83	—.07	18.047	8.177
7.39	.54	9.53	.85	4.94	.57	—.05	2.056	.776
10.66	.86	17.42	.75	2.25	.69	.06	3.539	1.834
10.66	1.59	15.24	.86	3.84	.94	.11	10.282	3.434
10.58	.86	13.50	.86	3.35	.83	.04	4.954	1.508
10.72	.69	11.89	1.16	2.13	1.36	—.08	8.739	6.762
10.50	.71	10.14	1.28	2.72	1.64	—.04	11.943	3.224
13.71	.81	17.47	.79	4.17	.67	—.04	3.401	2.139
13.51	.87	15.65	1.66	5.57	.96	—.03	11.044	3.380
13.70	.90	13.46	.94	2.24	1.36	.03	9.170	4.373
13.41	.59	11.54	.88	2.33	.69	—.03	2.862	2.615
13.50	.78	9.78	.89	3.47	.83	—.00	4.558	1.395
16.64	.45	17.52	.58	2.81	.45	—.02	.947	.895
16.60	.73	15.20	.46	1.51	.40	.00	1.072	1.405
16.34	.62	13.68	.45	3.84	.25	.04	.567	1.078
16.60	.49	11.92	1.28	4.09	.87	.06	4.371	3.181
16.62	.85	9.70	1.08	2.27	1.48	—.10	10.835	7.016
18.82	1.44	17.67	.78	2.30	.87	—.12	7.818	4.463
19.61	.61	15.70	.64	2.49	.65	—.11	2.011	1.495
19.40	.66	13.30	1.48	2.38	1.90	—.13	14.901	12.171
19.49	.55	11.62	.56	2.12	.50	—.02	1.218	.917
19.29	.87	9.86	.69	4.07	.54	.11	2.592	2.263

Focal parameter values obtained using the same model
as in the preceding table and assuming an indeterminacy
at the angles of 5° for emergence and 10° for the azimuth,
respectively.

TABLE 6

1º layer

4.59	3.86	4.10
4.20	4.24	4.20

2º layer

4.66	4.18	4.31
4.51	4.18	4.04

3º layer

4.74	4.33	4.33
4.33	4.33	4.33

Model used for subsequent focal parameter determination; the rectangles measure 7x6.5 km.

1st layer 0.5 km thickness
2nd layer 1.5 km   »
3rd layer 3.0 km   »
4th layer is 5.40 km/sec throughout. thickness is
5.0 km all speeds are in km/sec.

TABLE 7

X	Sx	Y	Sy	Z	Sz	To	V	C
7.51	.83	17.62	1.52	5.49	2.64	—.05	26.457	9.971
7.54	.62	15.37	1.09	4.56	1.20	—.23	6.506	3.947
7.41	82.	13.48	1.12	3.69	1.43	—.34	10.544	6.592
7.53	.78	12.15	1.72	3.22	1.43	—.32	15.326	7.941
7.50	.39	9.50	.81	5.81	.20	—.15	.507	.446
10.41	.81	16.91	1.26	4.40	2.84	—.09	23.209	9.994
10.74	1.57	15.19	1.12	4.75	2.17	—.19	30.534	7.757
10.33	.97	13.86	.75	3.74	2.17	—.21	12.626	9.354
10.81	.85	11.79	1.06	2.71	1.69	—.28	12.187	8.177
10.39	.70	9.90	.96	3.17	1.25	—.24	6.735	4.968
13.79	.96	16.72	1.27		1.51	—.04	14.800	7.733
13.83	1.60	15.45	1.22	4.08	1.83	—.23	28.519	11.348
13.53	1.24	13.12	.74	6.98	2.75	—.22	20.298	9.497
13.76	1.12	11.46	.99	3.73	1.92	—.23	17.094	6.393
13.09	.47	9.51	.61	3.79	1.93	—.30	4.499	3.524
16.33	.40	17.53	.58	4.51	.72	—.07	1.315	1.696
16.65	.84	15.24	.61	2.62	.84	—.10	3.434	2.748
16.35	.82	13.41	.47	1.56	.63	—.01	1.976	2.287
16.35	.50	11.86	1.34	3.39	1.05	—.14	5.578	3.741
16.86	1.10	9.91	1.28	5.54	2.55	—.30	28.775	11.296
19.62	1.11	17.75	1.25	3.99	1.33	—.27	14.768	5.902
19.13	.68	15.33	.53		.68	—.11	1.976	2.268
19.15	.94	13.58	1.20	4.40	3.61	—.23	32.664	13.664
19.48	.54	11.40	.38	2.22	.88	—.17	1.434	2.309
19.90	1.04	9.47	.82	5.16	.62	—.29	4.219	2.505

Focal parameter values obtained using the model shown
in table 6.

TABLE 8

x	y	z	t
8.7	18.4	1.0	.4
4.7	15.6	7.2	—.9
4.8	13.8	3.7	—.6
6.1	11.2	.1	.5
8.1	11.1	9.1	—.4
10.9	16.9	2.1	.1
10.9	14.7	1.8	.2
10.3	12.8	7.1	—.5
11.6	12.3	2.4	—.1
10.0	11.6	.9	.4
11.3	14.6	8.3	—.5
11.0	12.4	2.1	.5
12.3	15.3	5.8	—.3
15.3	10.7	7.7	—.9
14.5	9.2	.1	—.1
14.2	16.8	2.7	—.5
16.8	14.1	3.4	—.4
14.3	13.8	5.9	—.4
18.1	11.4	7.0	—.4
14.5	13.9	1.8	—.4
20.6	17.1	3.7	—.4
19.7	13.8	4.6	—.5
20.4	13.7	4.7	—.7
20.5	12.6	2.4	—.5
18.8	10.3	3.0	—.5

Hypocentre values obtained using the model shown in table
6 and considering only first arrivals.

Conclusions

An initial consideration may be made concerning the stability of the solutions obtained as the model varies.

This is very important for a local network in which the model plays an essential role.

Furthermore, the use of this type of method (which is in any case still to be optimized) to locate seismic events could make it possible to use the ratio between the quantity linked to ray convergence and the volume to evaluate the physical size of an earthquake source.

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